E-FIELD FROM AN ELECTRICALLY SHORT DIPOLE AS A FUNCTION OF VOLTAGE ON THE DIPOLE

(HANDOUT)

The θ component of the electric field from a small dipole ($L << \lambda$) with θ set to 90° (orthogonal direction to the dipole) is given by the following expression as a function of current on the dipole:

$$E_\theta = \frac{IL\sin\theta}{2\pi\varepsilon}\left(\frac{j\omega}{c^2 r} + \frac{1}{cr^2} + \frac{1}{j\omega r^3}\right)$$

Where:

I = Current in amperes
L = Length of dipole in meters
ε = 8.842×10^{-12}
θ = Angle to the plane of the dipole
r = Distance from the dipole in meters at which field is calculated
c = Speed of light in meters per second (3×10^8)
ω = Angular frequency of the signal exciting the dipole ($2\pi f$)

Since we are choosing the special case where θ = 90° this equation becomes:

1.) $$E_\theta = \frac{IL}{2\pi\varepsilon}\left(\frac{j\omega}{c^2 r} + \frac{1}{cr^2} + \frac{1}{j\omega r^3}\right)$$

Rewriting (1.) for the purpose of separating it into it's real and imaginary parts yields:

$$E_\theta = \frac{jIL}{2\pi\varepsilon}\left(\frac{\omega}{c^2 r} - \frac{1}{\omega r^3}\right) + \frac{IL}{2\pi\varepsilon}\left(\frac{1}{c r^2}\right)$$

or:

$$E_\theta = \frac{IL}{2\pi\varepsilon}\left(j\left[\left(\frac{\omega}{c^2 r} - \frac{1}{\omega r^3}\right)\right] + \frac{1}{c r^2}\right)$$

Therefore:

$$2.)\ |E_\theta| = \frac{IL}{2\pi\varepsilon}\sqrt{\left(\frac{\omega}{c^2 r} - \frac{1}{\omega r^3}\right)^2 + \left(\frac{1}{c r^2}\right)^2}$$

To get "I" in terms of volts impressed on the antenna, we need impedance "Z" of the antenna. This impedance has two components, a resistive (R_o) and an imaginary (X_o) component.

Therefore, the magnitude of the impedance "Z" may be expressed as:

$$3.)\ |Z| = \sqrt{R_o^2 + X_o^2}$$

And:

$$4.)\ R_o = 20\,[\beta L]^2 \text{ and } \beta = \frac{2\pi}{\lambda}$$

$$5.)\ X_o = \frac{30}{\beta L}\left[-4 + 4\ln\left(\frac{L}{A}\right)\right]$$

Where:

L= Length of dipole in meters

A= Diameter of the wire in meters

$\lambda = 3 \times 10^8 / f_{Hz}$

Substituting for β, (4.) becomes:

$$4.)\quad R_o = 20\left[\frac{2\pi}{\lambda}L\right]^2 = \frac{80\pi^2 L^2}{\lambda^2}$$

It is now possible to substitute for R_o and X_o in (3.) which yields:

$$6.)\quad |Z| = \sqrt{\left(\frac{80\pi^2 L^2}{\lambda^2}\right)^2 + \left(\frac{15\lambda}{\pi L}\left[-4 + 4\ln\left(\frac{L}{A}\right)\right]\right)^2}$$

Substituting V/Z for I in equation (2.) yields:

$$7.)\quad |E_\theta| = \frac{VL}{2\pi\varepsilon|Z|}\sqrt{\left(\frac{\omega}{c^2 r} - \frac{1}{\omega r^3}\right)^2 + \left(\frac{1}{cr^2}\right)^2}$$

Where V = Dipole volts RMS

Other observations:

For electrically short dipoles, the R_o term in the antenna impedance equation is usually several orders of magnitude less than the value of X_o.

If $r << \lambda$ and $\omega << c^2$ then (7.) becomes:

8.) $$E_{\theta} = \frac{V\, L}{2\, \pi \varepsilon |Z|\, \omega\, r^{3}}$$

And, $|Z| = X_o$

This simplified equation (8.) was used in the calculations presented for electric field emissions.

Example:

1 Volt RMS, 1 MHz on a dipole 0.1 meter long X 1 millimeter in diameter measured at a distance 1 meter from the antenna = 13.8 millivolts/meter field strength (83 dBμV/M)